Møller Energy-Momentum Complex for an Axially Symmetric Scalar Field

I. Radinschi*

Department of Physics, "Gh. Asachi" Technical University,
Iasi, 6600, Romania

Abstract

We calculate the energy-distribution for an axially symmetric scalar field in the Møller prescription. The total energy is given by the parameter m of the space-time.

Keywords: Møller energy-momentum complex, axially symmetric scalar field

PACS numbers: 04.20.-q

1 Introduction

The subject of energy-momentum localization in general relativity continues to be one of the most intricate because there is no given yet a generally accepted expression for the energy and momentum. It is well-known that various energy-momentum complexes [1]-[17] can give the same energy distribution for a given space-time. Recently, Virbhadra [10] studied if it is possible to obtain the same expression of the energy in the case of a Kerr-Schild metric by using the energy-momentum complexes of Einstein [18], Landau and Lifshtz [19], Papapetrou [20] and Weinberg [21] ELLPW. He concluded that these definitions lead to the same result. On the other hand, these definitions disagree for the most general nonstatic spherically symmetric metric [10]. Only the Einstein energy-momentum complex gives the same

---

*iradinsc@phys.tuiasi.ro

expression for the energy distribution when the calculations are performed in the Kerr-Schild Cartesian and Schwarzschild Cartesian coordinates.

Also, many results recently obtained, [16], [17], [22], [23] demonstrated that the Møller energy-momentum complex [24] is a good tool for obtaining the energy distribution of a given space-time. Møller energy-momentum complex allows to make the calculations in any coordinate system. In his recent paper, Lessner [25] concluded that the Møller definition is a powerful concept of energy and momentum in general relativity. Very interesting is the Cooperstock [26] hypothesis. He sustain that the energy and momentum are confined to the regions of nonvanishing energy-momentum tensor of the matter and all non-gravitational fields.

Also, Chang, Nester and Chen [9] showed that the energy-momentum complexes are actually quasilocal and legitimate expressions for the energy-momentum.

The purpose of this paper is to compute the energy distribution, for an axially symmetric solution that describes the space-time, which is endowed with a scalar field, in the Møller prescription. Recently gravitational lensing due to this space-time has been studied in a great detail [27], [28]. We use the geometrized units ($G = 1$, $c = 1$) and follow the convention that Latin indices run from 0 to 3.

2   Møller Energy-Momentum Complex for on Axially Symmetric Scalar Field

It is well-known that the Kaluza-Klein and the superstring theories predict the scalar fields as a fundamental interaction in physics. Scalar fields are fundamental components of the Brans-Dicke theory and of the inflationary models. Also, they are a good candidate for the dark matter in spiral galaxies. Because they interact very weakly with mater we have never seen one but many of the theories containing scalar fields are in good concordance with measurements in weak gravitational fields. Also, we expect that they can play an important role in strong gravitational fields like at the origin of the universe or in pulsars or black holes. The metric that we consider [27], [28] is an axially symmetric solution to the field equations derived from the action

for gravity minimally coupled to a scalar field. The solution is

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{e^{2k_a} dr^2}{1 - \frac{2m}{r}} - r^2 \left(e^{2k_a} d\theta^2 + \sin^2 \theta d\varphi^2\right), \qquad (1)$$

with

$$e^{2k_a} = \left(1 + \frac{m^2 \sin^2 \theta}{r^2 \left(1 - \frac{2m}{r}\right)}\right)^{-1/a^2}, \qquad (2)$$

$$\phi = \frac{1}{2a} \ln \left(1 - \frac{2m}{r}\right),$$

where a is a constant of integration and ϕ is the scalar field. This solution is one of the new classes of solutions to the Einstein-Maxwell theory non minimally coupled to a dilatonic field [28]. The metric given by (1) is almost spherically symmetric and represents a gravitational body (gravitational monopole) with scalar field. The scalar field deforms the spherically symmetry. We observe, that when $a \to \infty$, we recover the Schwarzschild solution. This metric can be employed to model the exterior field of a macroscopic object endowed with a minimally coupled scalar field, and it can be matched to a regular interior solution.

The Møller energy-momentum complex [24] is given by

$$\theta_i{}^k = \frac{1}{8\pi} \frac{\partial \chi_i{}^{kl}}{\partial x^l} \qquad (3)$$

where

$$\chi_i{}^{kl} = \sqrt{-g} \left(\frac{\partial g_{in}}{\partial x^m} - \frac{\partial g_{im}}{\partial x^n}\right) g^{km} g^{en}. \qquad (4)$$

The energy in the Møller prescription is given by

$$E = \iiint \theta_0{}^0 dx^1 dx^2 dx^3 = \frac{1}{8\pi} \iiint \frac{\partial \chi_0{}^0}{\partial \chi^l} dx^1 dx^2 dx^3. \qquad (5)$$

The Møller energy-momentum complex is not necessary to carry out the calculations in the quasi-Cartesian coordinates, so we can calculate in the spherical coordinates.

For the metric given by (1) the only required component of $\chi_0{}^{0l}$ is

$$\chi_0{}^{01} = 2m \sin \theta. \qquad (6)$$

Plugging (6) in (5) and applying the Gauss theorem we obtain

$$E = m. \tag{7}$$

The energy distribution is given by the mass m.

3 Discussion

We use an axially symmetric solution to the field equations for a scalar field minimally coupled to gravity. Virbhadra et al. [23] demonstrated that the presence of on object endowed with a scalar field acting as a gravitational lens would manifest through new lensing configurations. For the above situation we obtain the energy for the metric given by (1) by using the Møller energy-momentum complex. The energy distribution is given by the mass m. In the case of the Schwarzschild metric we obtain the same result. The Schwarzschild solution is a particular case of the metric that we use, when we have $a \to \infty$.

References

[1] K. S. Virbhadra, Phys. Rev. **D41,** 1086 (1990).

[2] K. S. Virbhadra, Phys. Rev. **D42,** 2919 (1990).

[3] N. Rosen and K. S. Virbhadra, Gen. Relativ. Gravit. **25,** 429 (1993).

[4] K. S. Virbhadra, Pramana - J. Phys. **45,** 215 (1995).

[5] A. Chamorro and K. S. Virbhadra, Pramana - J. Phys. **45,** 181 (1995).

[6] J. M. Aguirregabiria, A. Chamorro and K. S. Virbhadra, Gen. Relativ. Gravit. **28,** 1393 (1996).

[7] A. Chamorro and K. S. Virbhadra, Int. J. Mod. Phys. **D5,** 251 (1996).

[8] K. S. Virbhadra and J. C. Parikh, Phys. Lett. **B317,** 312 (1993); K. S. Virbhadra and J. C. Parikh, Phys. Lett. **B331,** 302 (1994).

[9] N. Rosen, Gen. Relativ. Gravit. **26**, 319 (1994); V. B. Johri, D. Kalligas, G. P. Singh and C. W. F. Everitt, Gen. Relativ. Gravit. **27** 313 (1995); N. Banerjee and S. Sen, Pramana – J. Phys. **49,** 609 (1997); I-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee, Int. J. Mod. Phys. **D6,** 349 (1997); I-Ching Yang, Wei-Fui Lin and Rue-Ron Hsu, Chin. J. Phys. **37**, 113 (1999), Chia-Chen Chang, J. M. Nester and Chiang-Mei Chen, Phys. Rev. Lett. **83,** 1897 (1999).

[10] K. S. Virbhadra, Phys. Rev. **D60,** 104041 (1999).

[11] S. S. Xulu, Int. J. Mod. Phys. **D7,** 773 (1998); S. S. Xulu, Int. J. Theor. Phys. **39**, 1153 (2000).

[12] S. S. Xulu, Int. J. Mod. Phys. **A15**, 4849 (2000).

[13] S. S. Xulu, gr-qc/0010062

[14] I. Radinschi, Mod. Phys. Lett. **A**, **15**, Nos. 11&12, 803 (2000).

[15] I. Radinschi, Acta Physica Slovaca, **49(5)**, 789 (1999).

[16] I. Radinschi, Mod. Phys. Lett. **A**, **15(35)**, 2171 (2000).

[17] I. Radinschi, Mod. Phys. Lett. A, 16(10), 673, (2001).

[18] C. Møller, Ann. Phys. *(*NY*)* **4**, 347 (1958); A. Trautman, in *Gravitation: an Introduction to Current Research*, ed. L. Witten (Wiley, New York, 1962, p. 169), R. P. Wallner, Acta Physica Austriaca **52**, 121 (1980)..

[19] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, 1987, p. 280).

[20] A. Papapetrou, Proc. R. Irish. Acad. **A52**, 11 (1948).

[21] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of General Theory of Relativity* (John Wiley and Sons, Inc., New York, 1972, p. 165).

[22] S. S. Xulu, gr-qc/0010062

[23] K. S. Virbhadra, D. Narasimha and S. M. Chitre, Astron.Astrophys. 337 (1998) 1-8, I-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee, Int. J. Mod. Phys. **D6,** 349 (1997); I-Ching Yang, Wei-Fui Lin and Rue-Ron Hsu, Chin. J. Phys. **37**, 113 (1999).

[24] C. Møller, Ann. Phys. (NY) **4,** 347 (1958).

[25] G. Lessner, Gen. Relativ. Gravit. **28**, 527 (1996).

[26] F. I. Cooperstock, Mod. Phys. Lett. **A14**, 1531 (1999).

[27] T. Matos, D. Núñez and H. Quevado, Phys. Rev. **D51**, 310, (1995).

[28] T. Matos, D. Nuñez and M. Rios, gr-qc/0008068.